**viscoelastic modeling of porcine ligaments**

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**Abstract.** Viscoelastic quasi-linear analytical models, as Fung, was implemented through the utilization of experimental results obtained from several porcine ligaments as: lateral collateral ligament (LCL), anterior cruciate ligament (ACL), posterior cruciate ligament (PCL) and medial collateral ligament (MCL). To implement quasi-linear viscoelastic models for soft tissues, as the Fung one, was necessary the utilization of a programming language, as C Sharp, and Object-oriented programming to deal with the model’s mathematical demands, as the convolution calculations. Moreover, those technologies allow to reduce the code execution time which was one of the main problems. Despite this benefit, was necessary to implement the numerical methods used in process. The models’ results show the stress evolution in relaxation tests. Although, the preliminary results show a good correlation between experimental and analytical models, showing a noticeable change in ligaments stiffness after the experimental implementation of relaxation tests.

**Keywords:** knee ligaments, analytic model, viscoelasticity, Fung

1. Introduction

The knee is one of the most complex joints of the body and is subjected to different loadings. The description of the mechanical behavior of the knee ligaments can be very useful to aid to model, quantitatively, the knee performance. Thus, researches have been published attempting to macroscopically analyze the knee ligaments/tendons through different viscoelastic mechanical models. (Rossetto, 2009) shows that this knowledge is important to better support the decisions in understand the physical training, such as in cases of therapy for tendinopathies. (Bernardes et. al, 2005) sought to determine the biomechanical parameters for modeling the human knee joint through extensive exercises, together with images obtained by videofluoroscope, where the viscoelasticity was accessed.

Viscoelasticity is understood as the property of materials that present viscous and elastic behavior at the same time, being a concept widely used in various sectors of the industry. The simplest viscoelastic model is one that considers linear functions, where the creep compliance and stress relaxation functions are depending only on time. This approach is commonly used for metals. (Tareco, 2014) uses Maxwell and Kelvin linear models to design a steel-concrete structure, analyzing the relaxation and creep compliance just for the concrete in the mixed structure response. Moreover, as presented by (Queiroz, 2008), viscoelastic materials are also used to attenuate vibrations and noise in structures, having application in both the automotive and aerospace sectors.

The quasi-linear viscoelastic model, proposed by (Fung, 1993), is commonly used in soft tissue research since it intends to describe its behavior close to reality. (Piazza et al., 2001) developed a three-dimensional dynamic model of the tibiofemoral and patellofemoral articulations to predict the knee implant movements during a step-up activity. They were based on the Fung’s model, using dynamic equations of motion subjected to forces generated by muscles, ligaments, and contact at articulations. Good results were achieved for the flexion-extension angle of the knee, but not for translations at the tibiofemoral articulations. (Debski et al, 2004) applied the Fung’s model and analyzed the viscoelastic properties of the healing goat medial collateral ligament, MCL. They characterized the reduced relaxation function and the elastic response and demonstrated that que quasi-linear viscoelastic model could be successfully used to describe the MCL viscoelastic behavior, during the healing phases.

Moreover, the quasi-linear viscoelastic method is frequently employed with computational resources since it has complex equations and not have any explicit analytical solution. (Xu and Engquist, 2018) proposed a mathematical model for relaxation modulus based on nonlinear model and its numerical solution and developed a finite-element framework and a numerical algorithm to implement this model for simulating responses under static and dynamic loadings. They validated the model through the utilization of various materials, comparing both experimental and numerical results. (Weiss et al, 2001) reviewed earlier and current techniques for the computational modeling of soft tissues, showing relevant concepts under the perspective of continuum mechanics and finite element. Also emphasized the microstructural influence of soft tissues. (Abramowitch et al., 2004) obtained the constants for quasi-linear viscoelastic model that are used to describe the elastic response, with constants *A* and *B*, and the reduced relaxation function, with constants *C*, and , together with an improved approach that converges to a single solution with minimal variation. They subjected six goat femur-medial collateral ligament-tibia to a uniaxial tensions test when consider ramp time. In tests, the convergence failed for three ligaments, with the biggest errors at constants *A*, *B* e .

The aim of this paper is to explain how to implement numerically the Fung’s quasi-linear viscoelasticity model. Using the C# programming language as in (Wagner et al, 2021) and ASP.NET MVC framework as in (Rick Anderson, 2019) and (Gasparotto, 2014). (Silveira, 2020) developed a REST API capable of performing the necessary calculations for this model and generating a CSV file to compare the numerical results with the experimental ones. The API was developed focusing on scalability, maintainability, and readability, applying some design patterns, as Strategy, and object-oriented programming patterns, as SOLID principles, and some resources were also used to optimize that software, as Swagger, for building the user interface. Finally, both numerical and experimental results was compared for each ligament.

1. Fung’s quasi-linear viscoelastic model

The quasi-linear viscoelastic model, proposed by (Fung, 1993), propose a non-linearity stress-strain relation, divided in two parts: the reduced relaxation function, which depends only on time, and elastic response, which depends on strain. This model is commonly used for soft tissue with good approximation. The constants needed for the equations are obtained experimentally. However, the Fung’s model has limitations, as for distinct relaxations and strain levels, different constants values are found.

* 1. Mathematical equation

(Fung, 1993) propose equations for elastic response, reduced relaxation function and stress considering one relaxation. For two relaxations considered in this paper, it is necessary to reformulate these equations. Moreover, each parameter will be expressed differently when considering and disregarding ramp time, except for reduced relaxation function.

* + 1. Strain

The equations used to describe the strain were developed to represent the experiments. When considering ramp time, the strain behavior is expressed by equation (1). When the strain maintains at the maximum value , it represents the relaxation. When stays at the minimum value , represents the recovery. That behavior also is observed in (Duenwald, et al., 2009). When disregard ramp time, the equation (2) is used, where is considered a constant strain during whole experiment.

|  |  |
| --- | --- |
| , | (1) |

where, the parameters and represent, respectively, ramp time and strain rate applied in experiment, with used when strain increase and , when it decreases. Furthermore, the parameters , , and are the time limits for each equation, indicating when the strain behavior changes.

|  |  |
| --- | --- |
| , | (2) |

where, represents the constant strain applied in the experiment.

It is possible to calculate the derivative that will be used in the stress calculations step. The equations (3) and (4) are the time derivative of, respectively, equations (1) and (2).

|  |  |
| --- | --- |
| , | (3) |
| . | (4) |

Gráfico

Descrição gerada automaticamente Gráfico, Gráfico de caixa estreita

Descrição gerada automaticamente

(a) (b)

Figure 1. (a) Strain per time and (b) strain derivative per time.

* + 1. Elastic response

The elastic response corresponds the soft tissue elastic part. As mentioned previously, two equations are used to describe the elastic response. When considering ramp time, an exponential approximation can be used like in research (Abramowitch, 2004).

|  |  |
| --- | --- |
| , | (5) |

where constants *A*, in Pa (Pascal), and *B*, dimensionless, are material constants and represents, respectively, elastic stress constant and elastic power constant. Moreover, as shown previously, the equation (5) can be rewritten with only time dependence.

|  |  |
| --- | --- |
| , | (6) |

where, is the initial stress applied in experiment.

The derivative for elastic response must be calculated because it will be used in equations for describe the stress. The derivative in time and in strain for equation (5):

|  |  |
| --- | --- |
| , | (7) |

|  |  |
| --- | --- |
| , | (8) |

When disregarding ramp time, the elastic response is considered constant for all time domain.

|  |  |
| --- | --- |
| , | (9) |

The derivative in time and in strain for equation (9):

|  |  |
| --- | --- |
| . | (10) |

* + 1. Reduced relaxation funcion

The reduced relaxation function represents the viscous portion and occurs for all time domain begging at 1, **g**(0) = 1. According with (Fung, 1993), it can be described in two ways. The first, equation (11), also called the simplified reduced relaxation function, is written as the Prony Series taking only three elements in the sum, in line with (Babaei et al, 2015), that affirmed that three elements were sufficient for a good approximation. Also, (Funk et al., 2000) stated that more than three elements do not result in significant gain. The second, equation (12), was developed from Kelvin model, standard linear solid (Fung, 1993), and uses integrals that only have numerical solutions. Moreover, both equations were implemented and tested but only the first was used, as constants are easier to be calculated experimentally.

|  |  |
| --- | --- |
| , | (11) |

where and are material dimensionless constants called relaxation modulus and represents the amplitude of the stress curve in relaxation, and is the relaxation time in seconds, also a material constant.

|  |  |
| --- | --- |
| , | (12) |

where *C*, and are material constants and represents, respectively, a dimensionless relaxation constant, fast and slow relaxation times in second. To improve the numerical implementation, that equation was rewritten as shown in Appendix, is obtained:

|  |  |
| --- | --- |
| , | (13) |

Also calculating the derivative in time for each equation (11) for reduced relaxation function, in Appendix:

Deriving equation (11):

|  |  |
| --- | --- |
| . | (14) |

* + 1. Stress

(Fung, 1993) shows three equivalent equations to calculate the stress:

|  |  |
| --- | --- |
| , | (15) |
| , | (16) |
| , | (17) |

As mentioned above, the elastic response and reduced relaxation function can be expressed only depending on time, so the partial derivative can be changed by total derivative. Moreover, and .

|  |  |
| --- | --- |
| , | (18) |
| , | (19) |
| . | (20) |

While considering ramp time, all equations return satisfactory results. Disregarding ramp time, the elastic response is constant, and its derivative is zero for all time domain, as shown previously. Thus, the equation (18) cannot be used, because it always returns zero since, and equations (19) and (20) can be rewritten.

As shown in Appendix presented, equations (19) and (20) return the same equation. So, when disregarding ramp time, a unique equation can be used.

|  |  |
| --- | --- |
| . | (21) |

Gráfico, Gráfico de caixa estreita

Descrição gerada automaticamente Gráfico, Histograma

Descrição gerada automaticamente

(a) (b)

Figure 2. Stress in MPa per time in second using (a) equation (18), (19) and equation (20) and (b) equation (21).

1. numerical implementation

The numerical implementation of Fung’s Model was developed by two steps: creating a class that represents the model and contains the equations for each parameter and its derivative in time; and creating a class to orchestrate the operation. Also was created an artificial frontier in the code that separate the operation and the model, being created specific contracts for each one. It was done based on Single-Responsibility Principle that gets easier to implement resources, prevents unexpected side-effects and improves maintainability. It is notable that the execution time lasted for minutes and, in worst cases, for hours, because the equations used to calculate the stress were not optimized for numeric applications. To improve their performance, it was used the class Task, a native resource from C#, with the aim to let some steps be processed asynchronous, executing multiple tasks together and reducing the execution time. It was used in both classes mentioned, in first, when calculating the results, and, in second, when iterating the input list, reducing that time to seconds, in worst case.

In Fig. 1 shows the flowchart for main operation that calculates the results for Fung’s model and the sub-routine The class that represents the model also contains a method, represented in Fig. 1 as sub-routine “Calculate Results”, that calculate in parallel all results necessaries - strain, elastic response, reduced relaxation function and stress – as shown in Fig. 1.b, and returns those values in an object. The orchestrator, as called, is responsible to orchestrate the operation, executing each step shown on Fig. 1.a, furthermore, previously the request data is validated to certain if it is valid and any error will be thrown during code execution.

Diagrama

Descrição gerada automaticamente

(a) (b)

Figure 3. Flowchart for (a) main operation and (b) sub-routine “Calculate Results”.

Also, it was necessary to implement numerical methods to deal with integrations and derivatives present in stress and reduced relaxation function equations. For the integrals, the Composite Simpson's Rule, equation (22), as in (Regra de Simpson, 2021) (Regras Compostas, 2021), was used. For the derivatives, the Symmetric Derivative, equation (23), (Da Cruz, 2012) was used since it gives the precision necessary while calculating the parameters.

|  |  |
| --- | --- |
| , | (22) |

where f(x) is an integrable function, a and b are the limits of integration, x is a differential of the variable x, and N is the number of subdivisions.

|  |  |
| --- | --- |
| , | (23) |

where f(x) is a differentiable function and x is a differential of the variable x.

1. Numerical extrapolation

Besides the numerical implementation of the quasi-linear viscoelastic model, for a better comparison with experimental results, it was necessary to develop a routine for extrapolating the experimental results. For implement this, it was necessary to predict the next values ​​based on the earlier stress curves' behavior. Taking into account, two important behaviors during relaxation: the stress decreases on time and the concavity is upwards. These typical behaviors were used to validate each point before extrapolation, to remove invalid points that may interfere in the final extrapolated results.

Diagrama

Descrição gerada automaticamente

Figure 4. Flowchart for numerical extrapolation.

The numerical extrapolation was made according to flowchart at Fig. 2, it is noteworthy that after the API receives input data, these are validated to ensure that the file has enough lines for the operation and the parameters that were passed are correct. The operation was divided in two subroutines to improve maintainability and readability, since the software may be used in future research.

1. RESULTS AND CONCLUSIONS

Mostrar os resultados numéricos sem o tempo de subida.

1. References

Silveira, B. M. 2020. “SoftTissue”. Available in: https://github.com/M3110/SoftTissue. Accessed on June 01, 2021.

Fung, Y. 1993. “Biomechanics: Mechanical Properties of Living Tissues”. Springer, New York, University of Michigan.

Duenwald, S. E.; Jr., R. V.; Lakes, R. S., 2009. “Viscoelastic Relaxation and Recovery of Tendon”. University of Wisconsin-Madison. Madison, USA.

Abramowitch, Steven D. e Woo, Savio L.-Y., 2004. “An Improved Method to Analyze the Stress Relaxation of Ligaments Following a Finite Ramp Time Based on the Quasi-Linear Viscoelastic Theory”. JOURNAL OF BIOMECHANICAL ENGINEERING. ASME. Vol. 126. P. 92-97. DOI: 10.1115/1.1645528.

Funk, J. R., Hall, G. W., Crandall, J. R., Pilkey, W. D., 2000. “Linear and quasi-linear viscoelastic characterization of ankle ligaments”. Journal of Biomechanical Engineering, Vol. 122, No. 1, p. 15-22. DOI:10.1115/1.429623

Regra de Simpson. Instituto Superior Técnico de Lisboa. Available in: https://www.math.tecnico.ulisboa.pt/ ~calves/courses/integra/capiii33.html#:~:text=Regra%20de%20Simpson%20aplicada%20a%20dois%20sub%2Dintervalos.&text=Assim%2C%20podemos%20considerar%20tr%C3%AAs%20n%C3%B3s,cada%20um%20destes%20sub%2Dintervalos. Accessed on: May 18, 2021.

Regras Compostas. Universidade Federal do Rio Grande do Sul. Available in: https://www.ufrgs.br/reamat/ CalculoNumerico/livro-oct/in-regras\_compostas.html. Accessed on May 18, 2021.

Babaei, Behzad, et al. 2015. “Efficient and optimized identification of generalized Maxwell viscoelastic relaxation spectra”. Available in: http://dx.doi.org/10.1016/j.jmbbm.2015.10.008. Accessed on May 6, 2021.

Da Cruz, A. M.C. B., Martins, N., Torres, D. F.M. 2012. “Symmetric differentiation on time scales”. DOI:10.1016/j.aml.2012.09.005

Wagner, B., et al. 2021. “A tour of the C# language”. Microsoft. Available in: https://docs.microsoft.com/en-us/dotnet/csharp/tour-of-csharp/. Accessed on: June 14, 2021.

Tareco, M. A. C. 2014. Conceitos de viscoelasticidade na modelação da fluência em estruturas mistas aço-betão. 154f. Dissertação (Mestrado) – Engenharia Civil, Faculdade de Ciências e Tecnologia. Lisboa, 2014. Available in: https://run.unl.pt/bitstream/10362/12481/1/Tareco\_2014.pdf. Accessed on May 4, 2021.

Queiroz, José Aparecido Silva de. 2008. “Flexible structures analysis with viscoelastic materials application”. Universidade Estadual Paulista, Faculdade de Engenharia de Bauru, 2008. Available in: https://repositorioslatinoamericanos.uchile.cl/handle/2250/2568006. Accessed on May 4, 2021.

Rossetto, N. P. 2009. “Viscosity in stretching tendons”. Universidade Estadual de Campinas. Campinas.

Bernardes, C., et al. 2005. “Biomechanical parameters' determination for knee joint modeling”. Laboratório de Pesquisa do Exercício, Universidade Federal do Rio Grande do Sul. Porto Alegre.

Zheng, N.; et al. 1998. “An analytical model of knee for estimation of internal forces during exercise”. American Sports Medicine Institute. Birmingham, Alabama, USA.

Rick Anderson. 2019. “ASP.NET overview”. Microsoft. Available in: https://docs.microsoft.com/en-us/aspnet/overview. Accessed on June 19, 2021.

Gasparotto, H. M. 2014. “ASP.NET MVC Introduction”. DEVMEDIA. 2014. Available in: devmedia.com.br/introducao-ao-asp-net-mvc/31878. Accessed on June 19, 2021.

Piazza, Stephen J.; Delp, Scott L. 2001. “Three-Dimensional Dynamic Simulation of Total Knee Replacement Motion During a Step-Up Task”. Journal of Biomechanical Engineering. ASME. Vol. 123. P. 599-606. DOI: 10.1115/1.1406950.

Debski, R. E; Abramowitch, S. D.; Woo, S. L.-Y.; Clineff, T. D. 2004. “An evaluation of the quasi-linear viscoelastic properties of the healing medial collateral ligament in a goat model”. Annals of Biomedical Engineering. DOI: 10.1023/b:abme.0000017539.85245.6a

Xu, Qinwu e Engquist, Björn. 2018. “A mathematical model for fitting and predicting relaxation modulus and simulating viscoelastic responses”. DOI: 10.6084/m9.figshare.c.4088969.

Weiss, Jeffrey A. e Gardiner, John C. 2001. “Computational Modeling of Ligament Mechanics”. University of Utah, Department of Bioengineering, Salt Lake City, Utah 84112.

De Pascalis, R.; Abrahams, I.D.; Parnell, W.J.. 2014. “On nonlinear viscoelastic deformations: a reappraisal of Fung’s quasi-linear viscoelastic model”. DOI: 10.1098/rspa.2014.0058.

7. APPENDIX

Based on material properties and the constants definition, can be assumed that , so, , therefore, could be rewritten like:

|  |  |
| --- | --- |
| , | (i) |

Then:

|  |  |
| --- | --- |
| , | (ii) |

Applying equation (ii) in (12):

|  |  |
| --- | --- |
| , | (13) |

Deriving (13) in function of time:

|  |  |
| --- | --- |
| , | (iii) |

Applying the definition of calculus to the derivative of a definite integral:

|  |  |
| --- | --- |
| , | (iv) |

where , and .

|  |  |
| --- | --- |
| , | (v) |

Applying (v) in (iii):

|  |  |
| --- | --- |
| , | (14) |

Rewriting (19):

|  |  |
| --- | --- |
| , |  |
| , |  |
| , |  |
| , |  |
| . | (21) |

Rewriting (20):

|  |  |
| --- | --- |
| , |  |
| , |  |
| . | (21) |

8. Responsibility notice

The authors are the only responsible for the printed material included in this paper.